PAPIER-HANDLUNG

Dr. Gottlieb Adler

Hydrostatik u. Hydrodynamik

II. S. 91. M. Smoluchowski

F. POLLY, IV. KAROLINENG. 23.

[illegible]



[illegible] Will. Thomson [illegible]

[illegible]

Hydrostatik

Capillaritätstheorie

Hydrodynamische [illegible] [Euler]

[illegible] Hydrodynamik: [illegible] [Helmholtz]

[illegible]

[illegible] math. [illegible]

[illegible]

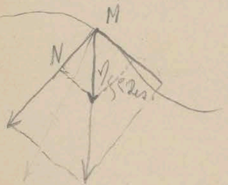
Bjerknes [illegible] 2 [illegible] (2)

[illegible]

[illegible]

[illegible]

1. [illegible] [[illegible]]



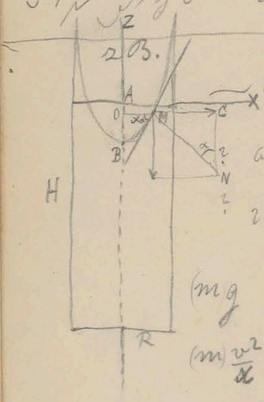
[illegible]

[illegible] comp. [illegible] tg. [illegible]

[illegible]

[illegible] tg. comp. [illegible] Res. [illegible]

[illegible] ⊥ [illegible] Res.



Cyl. rotation [illegible]

[illegible] Curve

[illegible]

$(mg \quad \left(\frac{n}{m=1}\right)$

$(m)\frac{v^2}{x} \qquad v = \frac{2x\pi}{T}$

$= \frac{4\pi^2}{T^2}\cdot x$

$\frac{dz}{dx} = \operatorname{tg}\alpha = \frac{mc}{g} = \frac{4\pi^2}{gT^2}x \qquad z = \frac{2\pi^2}{gT^2}x^2 + K$

∴ Parabel

$x=0 \mid z = K \qquad K = OB$

? [illegible] ?

AB = ?

[illegible] Vol. [illegible]

$\pi r^2 H$



$$\vartheta = 2\pi \int_{x=0}^{x=r} z\, x\, dx = 2\pi \int_0^r \left[K x + \frac{2\pi^2}{g^T} x^3\right] dx =$$

$2\pi x\, dx \cdot z$



$$= 2\pi \left[\frac{x^2}{2} + \frac{2\pi^2}{g^T} \cdot \frac{x^4}{4}\right]_0^r =$$

$$= K r^2 \pi + \frac{4\pi^3}{g^T} \frac{r^4}{4}$$

$$K r^2 \pi + \frac{\pi^3 r^4}{g^T} = \pi r^2 H$$

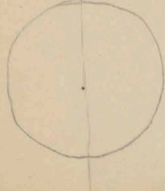
$$K + \frac{\pi^2 r^2}{g^T} = H$$

$$\underbrace{H - K}_{AB} = \frac{\pi^2 r^2}{g^T}$$

$$AB = \frac{\pi^2 r^2}{g^T}$$

[illegible] T [illegible]

2OB.

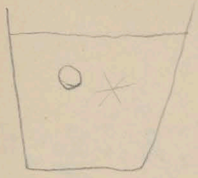


[illegible]

[illegible]

[illegible]

2).? [illegible] [[illegible]]



[illegible] 1) [illegible]; [illegible] etc.

2) [illegible] Res. [illegible]

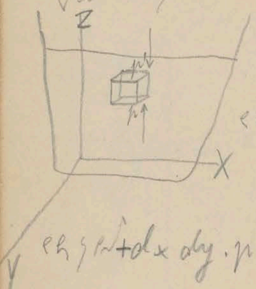
[illegible] [molecular [illegible]]

[illegible]

[illegible]

[illegible]

[illegible]



[illegible] parallelepiped [illegible]

[illegible] $p = f(x,y,z)$ [[illegible]]

[illegible] $+dx\,dy \cdot p \qquad - p'\,dx\,dy$

$$dx\,dy\,[p - p']$$

$$p = f(x,y,z)$$

$$p' = f(x,y,z+dz) = f(x,y,z) + \frac{\partial f}{\partial z}\,dz$$

$$= p + \frac{\partial p}{\partial z}\,dz$$

$$p - p' = -\frac{\partial p}{\partial z}\,dz$$

$$Z = -\frac{\partial p}{\partial z}\,dx\,dy\,dz$$

$X = -\frac{\partial p}{\partial x}\,dx\,dy\,dz$

$Y = -\frac{\partial p}{\partial y}\,dx\,dy\,dz$

[illegible]

[illegible] …

$\rho\,dx\,dy\,dz =$ [illegible]

$X\ Y\ Z =$ … [illegible] [illegible g]

$$X\rho\,dx\,dy\,dz - \frac{\partial p}{\partial x}\,dx\,dy\,dz = 0$$

$$\left.\begin{aligned}\frac{\partial p}{\partial x} &= \rho X\\ \frac{\partial p}{\partial y} &= \rho Y\\ \frac{\partial p}{\partial z} &= \rho Z\end{aligned}\right\}$$ [illegible] H

[illegible]

$X = 0 \quad Y = 0 \quad Z = g$ [illegible].

$\frac{\partial p}{\partial x} = 0$ [illegible] XY [illegible] constant

$\frac{\partial p}{\partial y} = 0$ [illegible] Niveauflächen

$\frac{\partial p}{\partial z} = \rho g$ $\quad p = \rho g z + K \quad K$ [illegible] $z = 0$

[illegible] Coord. [illegible] atm [illegible]

$Z = p h s$

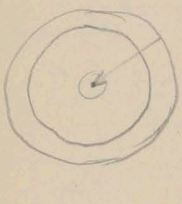
[illegible]

[illegible] Niveauflächen concentrische [illegible]

[illegible] r variabel

$\frac{\partial p}{\partial r} = \rho P$

[illegible] negativ [illegible]



[illegible]

$$P = \frac{\frac{4}{3}\pi r^3 \cdot \rho}{r^2} = \frac{4\pi}{3} r \rho$$

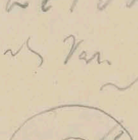
$$\frac{4\pi\rho}{3} R = g$$

$$P = \frac{g r}{R}$$

$-\frac{\partial p}{\partial r} = \frac{\rho g}{R} r$ [illegible]

[illegible] Var. [illegible] : $y =$ [illegible]

$\frac{\partial p}{\partial y} = \frac{\rho g}{R}(R-y)$



$\partial y = -\partial r$

$$p = \frac{\rho g}{R}\left[Ry - \frac{y^2}{2}\right]$$

[illegible] : $p = \frac{\rho g}{R}\frac{R^2}{2} = \rho g \frac{R}{2}$

[illegible] 150.000 Atm

[illegible] Compressibilitäts-coeff [illegible] $\frac{1}{20.000}$ [illegible]

sp. Gew. 8

[illegible] sp. G. [illegible] = 5'4 [illegible]

$$\frac{\partial p}{\partial x} = \rho X \quad \Big| \; dx$$

$$\text{---} \quad \Big| \; dy$$

$$\text{---} \quad \Big| \; dz$$

$$dp = \rho(X dx + Y dy + Z dz)$$ [illegible]

$$X = \frac{\partial V}{\partial x} \quad Y = \frac{\partial V}{\partial y} \quad Z = \frac{\partial V}{\partial z}$$

V = Potential

$$dp = \rho\Big(\underbrace{\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz}_{\text{Diff. } V}\Big)$$

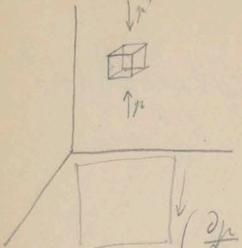
$$dp = \rho\, dV$$

V [illegible] const. [illegible] $dp = 0$ [illegible] $dV = 0$

[illegible] Poten. [illegible]

[illegible] Pot. [illegible] Potn. [illegible] compressibilität; [illegible] inc. [illegible]

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$p \quad p'$

$p' = p + \frac{\partial p}{\partial z} dz$

$p - p' = -\frac{\partial p}{\partial z}\, dz\, dx\, dy = Z\, dx\, dy\, dz$

$$\text{I} \begin{cases} \frac{\partial p}{\partial x} = \rho X \\ \frac{\partial p}{\partial y} = \rho Y \\ \frac{\partial p}{\partial z} = \rho Z \end{cases}$$

[illegible] $X = 0 \quad Y = 0 \quad Z = +g$

$\frac{\partial p}{\partial z} = \rho g \quad \rho$ const.

$p = \rho g z + A \quad \Big|_{p = A}^{z = 0}$

[illegible] Niveau [illegible]

ρ [illegible] constant [illegible]

[illegible] [[illegible]]

$\frac{\partial p}{\partial z} = \rho g \quad \rho$ [illegible] constant [illegible]

[illegible] p [illegible]

van der Waals [illegible]

[illegible] Mariotte'sche Gesetz $\rho : p = \rho_0 : p_0$

$p_0 = 1033\, g \qquad \rho = \frac{\rho_0}{p_0} p$ [illegible]

$\rho_0 = \frac{1}{777}$ [illegible] tempe[illegible]

$\frac{\partial p}{\partial z} =$ [illegible] Coord. [illegible] Niveau [illegible]

[illegible]

[illegible]

$$\frac{\partial p}{\partial z} = -g \frac{\rho_0}{p_0} p$$

$$\frac{\partial p}{p} = -g \frac{\rho_0}{p_0} \partial z$$

$$\log p = -g \frac{\rho_0}{p_0} z + \log A$$

$$p = A e^{-g \frac{\rho_0}{p_0} z}$$

$$z = 0 \mid p = p_1$$

$$p = A = p_1$$

$p_1 =$ [illegible]

$$p = p_1 e^{-g \frac{\rho_1}{p_0} z}$$

$$\log p = -g \frac{\rho_0}{p_0} z + \log p_1$$

$$z = \frac{\log p_1 - \log p}{g \frac{\rho_0}{p_0}} = \frac{p_0}{g \rho_0} \left[\log p_1 - \log p\right]$$

$$\frac{1033 \; cm^3}{1 cm^2 \; \frac{980 cm}{sec^2} \; 777} = 777000 cm = 7770 m$$

[illegible] cm^2 [illegible] temp. 0)

$$z = 18350 \left[\log p_1 - \log p\right]$$ Formel f. d. barometrische Höhenmessung

Capillarität

[illegible] Capillaritätstth. [illegible] Lapl. Th. 18

Gauss [illegible]

[illegible]

2). [illegible]

[illegible]

$A = \alpha(F' - F)$ [illegible] prop. [illegible]

$B = \beta(S' - S)$ [illegible] Potenz

[illegible] Gauss [illegible]

Princip d. virt. Verschiebungen [v. Bernoulli]

[[illegible] Duhamel Zirkel]

[illegible] Energie.

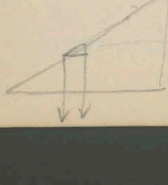
[illegible] Σ [illegible]

[illegible] Z, k, [illegible] Sek.

[illegible] P. d. v. V. [illegible]

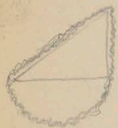
$X \delta x$ ✓ = Prod. [illegible]

[illegible] Σ [illegible] ✓



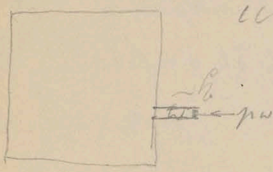
Stevin [illegible]

[illegible]





[illegible]



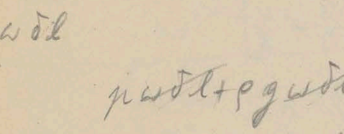
[illegible] $pw\,\delta l$ = [illegible] Niveau [illegible]

$\rho g w\,\delta l$

$$p w \delta l + \rho g w \delta l z = 0$$

$$p = -g\rho z$$

oder hydraul. Presse



$$a\delta h + A\delta H = 0$$

$$f\delta h = F\delta H$$

$$\delta H = \frac{f}{F}\delta h$$

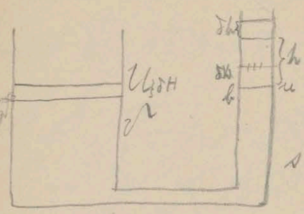
$$a\delta h + A\frac{f}{F}\delta h = 0$$

$$A = \frac{aF}{f}$$

$\beta(\mathfrak{z}' - \mathfrak{z})$

[illegible] [qual.] [illegible]

[illegible] [quant.]



$s = \rho g$

$$s b \delta h . h + \beta u \delta h - \beta U \delta H = 0$$

$$B \delta H = b \delta h$$

$$\delta H = \frac{b}{B} \delta h$$

[illegible] δH [illegible] δh [illegible]

[illegible] continuitäts [illegible] | [illegible]

$$s b h \delta h + u \beta \delta h - \beta U \frac{b}{B} \delta h = 0$$

$: b \delta h$

$$s h + \beta \frac{u}{b} - \frac{U}{B} \beta = 0$$

$$h = \frac{\beta}{s} \left[\frac{U}{B} - \frac{u}{b} \right]$$

[illegible] $U = 2R\pi$ $u = 2r\pi$

$B = R^2\pi$ $b = r^2\pi$

$$h = \frac{\beta}{s} \left[\frac{2}{R} - \frac{2}{r} \right] = \frac{2\beta}{s} \left[\frac{1}{R} - \frac{1}{r} \right] \quad R > r$$

$\beta > 0$

$h < 0$

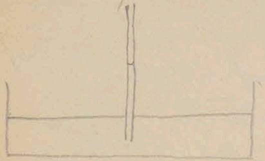
$W = \beta (B' - B)$

[illegible]

sp. Hg

[illegible]

[illegible]



$h = -\frac{2\beta}{r s}$ [illegible] prop. [illegible] Rad.

[illegible] quadr. [illegible]

$U = 4A$ $u = 4a$

$B = A^2$ $b = a^2$

$h = \frac{4\beta}{s}\left[\frac{1}{A} - \frac{1}{a}\right]$

[illegible] 2 Planpar. Platten

$U = 2L$ $B = LD$

$u = 2l$ $b = ld$

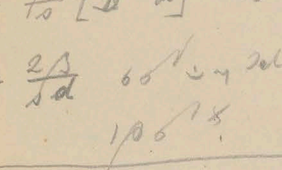
$h = \frac{2\beta}{s}\left[\frac{1}{D} - \frac{1}{d}\right]$ $D = \infty$

$h = \frac{2\beta}{sd}$ [illegible]

$A = \alpha\left[F' - F\right]$ [illegible] Langley [illegible]

[illegible]

[illegible]



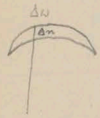
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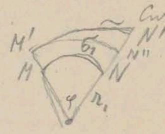
[illegible] α F [illegible] F′ – [illegible]

$W = \alpha (F' - F)$ α – pos. const. [illegible] [Capien]

? [illegible]

Curv. elem. r_1 = [illegible] rad.





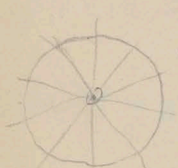
$\sigma_1 = r_1 \varphi$ MN

$\sigma_1' = (r_1 + \Delta n)\varphi$ MN''

$M'N' = \dfrac{N'N''}{\cos\alpha} = \dfrac{N'N''}{1 - \underbrace{\frac{\alpha^2}{1.2}}_{\text{[illegible] II [illegible]}}}$

[illegible]

$M'N' = M'N'' = (r_1 + \Delta n)\varphi$



[illegible] Normale [illegible] N [illegible] Normale [illegible]

[illegible] Normale [illegible] [[illegible]]

[illegible] 2 [illegible] $\perp \sigma$

[illegible] □ [α Gauss [illegible]]



$dw = \sigma_1 \sigma_2$

$dw' = \sigma_1' \sigma_2'$

$\sigma_1' = (r_1 + \Delta n)\varphi = (r_1 + \Delta n)\dfrac{\sigma_1}{r_1}$

$\sigma_2' = (r_2 + \Delta n)\varphi = (r_2 + \Delta n)\dfrac{\sigma_2}{r_2}$

$dw' = \left(\sigma_1 + \dfrac{\Delta n\, \sigma_1}{r_1}\right)\left(\sigma_2 + \dfrac{\Delta n\, \sigma_2}{r_2}\right) = \sigma_1\sigma_2 + \Delta n\, \sigma_1\sigma_2\left(\dfrac{1}{r_1} + \dfrac{1}{r_2}\right) + 0$

[illegible] Norm. [illegible]

$\sigma_1 \sigma_2 \Delta n \left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

[illegible] [α 20]

$\alpha \int \left(\frac{1}{r_1} + \frac{1}{r_2}\right) d\omega \, dn$

[illegible]

[illegible] $\sum p \, d\omega \, dn$

$\alpha \sum \left(\frac{1}{r_1} + \frac{1}{r_2}\right) d\omega \, dn + \sum p \, d\omega \, dn = 0$

$p = \alpha \left(\frac{1}{r_1} + \frac{1}{r_2}\right) = 0$

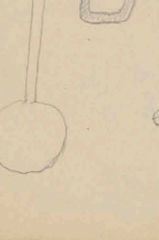
$\frac{1}{r_1} + \frac{1}{r_2} = \text{const.} = \frac{2}{R}$

[illegible]

$p + \frac{2\alpha}{R} = 0$

$p = 0 \quad R = \infty$

Plateau [illegible]

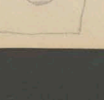


$pR = 2\alpha$

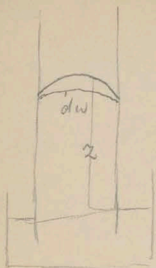
Plateau [illegible]

[illegible]

[illegible] Plateau [illegible]



s = sp. g.



$s\, dw\, \delta n . z$

$$\alpha \sum \left(\frac{1}{r_1}+\frac{1}{r_2}\right) dw\, \delta n + \sum s.z\, dw\, \delta n = 0$$

$$\alpha\left(\frac{1}{r_1}+\frac{1}{r_2}\right) + s.z = 0$$

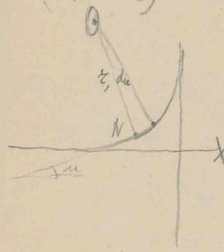
[illegible] conv. [illegible] z – [illegible] conc. +

$r_1 = r_2 = \infty \,|\, z = 0$; [illegible]

conv. [illegible] rad. +

$$\alpha\left(\frac{1}{r_1}+\frac{1}{r_2}\right) + s.z = 0$$

conc. –



? 1) [illegible] Curve

2). [illegible]

3). [illegible]

[illegible] ⊥ [illegible] ∞ [illegible] ; – √ [illegible]

$r_2 = \infty$

MN = $d\sigma = r_1\, du$

$dx = d\sigma \cos u$

$dz = d\sigma \sin u$

$$\frac{1}{r_1} = \frac{du}{d\sigma} \qquad \alpha \frac{du}{d\sigma} = s z$$

$$\alpha \frac{\sin u\, du}{dz} = s z$$

$$-\alpha \cos u = s\frac{z^2}{2} + C$$

$$z=0 \mid u=0 \qquad C=-\alpha$$

$$\alpha(1-\cos u) = s\frac{z^2}{2}$$

* Analine [illegible] = 90°

$$\alpha(1-\cos u) = s\frac{Z^2}{2}$$

$$\alpha = s\frac{Z^2}{2}$$

$$Z=\sqrt{\frac{2\alpha}{s}}$$ [illegible]

[illegible]

P const. [illegible]

$$\frac{s z^2}{2\alpha} = 1-\cos u$$

$$\cos u = 1 - \frac{sz^2}{2\alpha}$$

$$tg^2 u = \sec^2 u - 1 = \frac{1}{\left(1-\frac{sz^2}{2\alpha}\right)^2} - 1$$

$$\left(\frac{dz}{dx}\right) = \sqrt{\frac{1}{\left(1-\frac{sz^2}{2\alpha}\right)^2}-1}$$

$$a^2 = \frac{\alpha}{s}$$

$$x = \sqrt{4a^2-z^2} - a\log\sqrt{4a^2+z^2} + \frac{2a}{2}$$

[illegible]

pro [illegible]

$\mathfrak{z} = s\int z\,dx$

$= s\int z\,dz\,\frac{dx}{dz} = s\int \alpha \sin u\,du\,\frac{\cos u}{\sin u}$

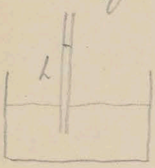
$= s\alpha \sin u\Big|_0^U = s\alpha \sin U$

[illegible] $U = 90°$ [illegible] $\mathfrak{z} = s\alpha$

[illegible]

$\underbrace{119{,}8 - 124{,}2}$ [illegible]

Quincke [illegible] Subst. $\frac{\alpha}{2s} = 4{\cdot}3\,\alpha$



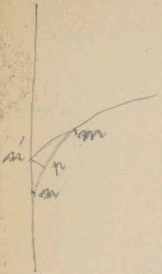
$\beta(\mathfrak{z}' - \mathfrak{z})$ $\quad \delta h$

$\beta\, 2\pi r\,\delta h + \pi r^2\,\delta h\, s\, h = 0$

$\frac{2\beta}{r} = -hs$

$h = -\frac{2\beta}{s}\frac{1}{r}$

[illegible]



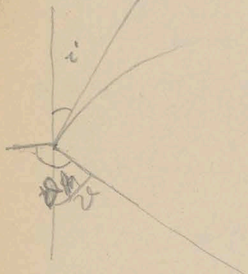
$\alpha(\underbrace{mn - mn'}) + \beta\, mn' + \text{const} \ldots = 0$

$np = nn' \cos i$

$-\alpha\, nn' \cos i + mn'\beta = 0$

$\beta = \alpha \cos i$

$\beta = -\alpha \cos \vartheta$



$\vartheta + i$ [illegible]

$h = -\dfrac{2\beta}{s r} = +\dfrac{2\alpha\cos\vartheta}{s r}$

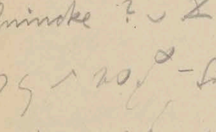
[illegible] ok.

[illegible] 2 const. [illegible] α [illegible] β [illegible] α [illegible] ϑ.

$h = \dfrac{2\alpha\cos\vartheta}{s}\, d$ [illegible]

Quincke [illegible] ϑ [illegible] Substanz [illegible]

[illegible] Silber [illegible]



[illegible] 20 [illegible]

[illegible]

Ag

[illegible] Ag [illegible] 13.7 mm

5 mm [illegible] 12 mm

6.5 " " 9.8 " 0.000054 mm

Quincke [illegible] Ag [illegible]

[illegible] Ag

[illegible] Molec. [illegible] sph. [illegible]

< 0'005 mm [illegible] etc.

Problem [illegible] Nominal [illegible] [Dr. Schwarz [illegible] Götting]

[illegible]

23/5 mv [illegible]

$m\frac{dv}{dt} = P$ Newton'sche [illegible]

Euler, [illegible] Lagrange [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

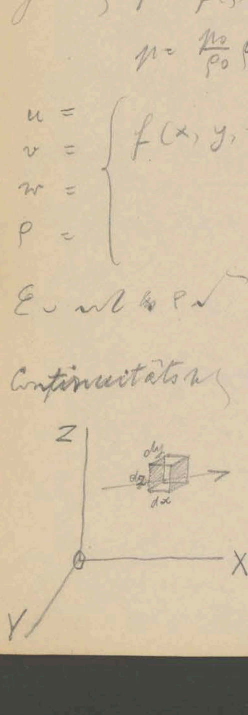
[illegible] $p = f(\rho)$ [illegible]

$p = \frac{p_0}{\rho_0}\rho$ } [illegible]

$$\left.\begin{array}{l} u = \\ v = \\ w = \\ \rho = \end{array}\right\} f(x, y, z, t)$$

[illegible]

Continuitäts[illegible] [[illegible] ponder. [illegible] imponder. [illegible]]



statist. [illegible]

? [illegible]

M = [illegible] Vol. [illegible]

$\rho\, dx\, dy\, dz$

$\rho + \frac{\partial \rho}{\partial t}$

$\left(\rho + \frac{\partial \rho}{\partial t} dt\right) dx\, dy\, dz$ = [illegible] $t + dt$

$\frac{\partial \rho}{\partial t} dt\, dx\, dy\, dz =$ [illegible]

[illegible]

[illegible] etc.

[illegible] Contin. [illegible]

[illegible] Materie.

[illegible] comp.

[illegible] $dy . dz .\ u . dt . \rho$ } [illegible] Prisma [illegible]

$dy\, dz\, dt (\rho u - \rho u') =$ [illegible] $\qquad \rho u = f(x, y, z, t)$

$(\rho u)' = f(x + dx, y, z, t)$

$- dy\, dz\, dt \frac{\partial(\rho u)}{\partial x} dx$

$= f(x, y, z, t) + \frac{\partial f}{\partial x} dx +$

$- dx\, dy\, dz\, dt \frac{\partial(\rho u)}{\partial x} =$ [illegible]

[illegible] 2 [illegible] $\qquad dy\, dz\, dt\, w \rho$

$- dx\, dy\, dz\, dt \frac{\partial(\rho w)}{\partial z}$

[illegible]: $\frac{\partial \rho}{\partial t}\, dx\, dy\, dz\, dt = -\frac{\partial \rho u}{\partial x}\, dx\, dy\, dz\, dt$

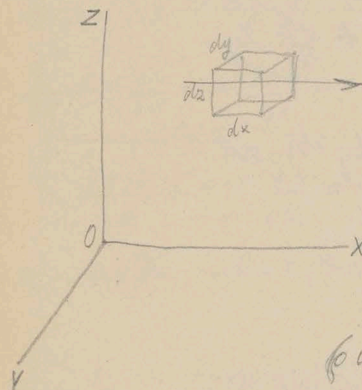
$$-\frac{\partial \rho v}{\partial y} \;\; - -$$

$$-\frac{\partial \rho w}{\partial z}\, dx\, dy\, dz\, dt$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

Continuitätsgl. [illegible]

[illegible] Subst. [illegible]



$m u$

$m v$

$m w$

[illegible] $\Sigma \rho$

[illegible]

$dx\, dy\, dz\, \rho u$

$\left(\rho u + \frac{\partial (\rho u)}{\partial t} dt\right) dx\, dy\, dz$

[illegible] $\frac{\partial \rho u}{\partial t}\, dx\, dy\, dz\, dt$

[illegible]

[illegible]

$dy\, dz\, u^2 dt . \rho . \qquad \rho u^2 dy\, dz\, dt$

[illegible]

$- dy\,dz\,dt\,\rho u'^2$

$\rho u^2 = f(x\,y\,z\,t)$

$\rho u'^2 = f(x+dx,\, y,\, z,\, t) = f + \frac{\partial f}{\partial x} dx$

$\rho u^2 - (\rho u)'^2 = - \frac{\partial(\rho u^2)}{\partial x} dx$

(a) $- \frac{\partial(\rho u^3)}{\partial x} dx\,dy\,dz\,dt$

~~dxdz~~ $\rho v u\, dx\,dz\,dt$

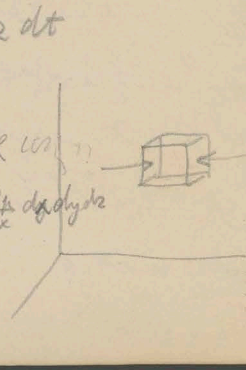
$\rho v u' = f + \frac{\partial f}{\partial y} dy$

(b) $- \frac{\partial(\rho u v)}{\partial y} dx\,dy\,dz\,dt$

$\rho\, dx\,dy\, w\, dt \,.\, u$

(c) $- \frac{\partial(\rho u w)}{\partial z} dx\,dy\,dz\,dt$

$\rho\, dx\,dy\,dz\, X\, dt$ z e ecos



$p\, dy\,dz - p'\, dy\,dz = -\frac{\partial p}{\partial x} dx\,dy\,dz$

$p' = p + \frac{\partial p}{\partial x} dx$

$-\frac{\partial p}{\partial x}\, dx\, dy\, dz\, dt$ [illegible]

[illegible] 1). [illegible] 2). [illegible]

$$-\frac{\partial(\rho u^2)}{\partial x} - \frac{\partial(\rho u v)}{\partial y} - \frac{\partial(\rho u w)}{\partial z} + \rho X - \frac{\partial p}{\partial x}$$

$$\underbrace{\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho u v)}{\partial y} + \frac{\partial(\rho u w)}{\partial z} = \rho X - \frac{\partial p}{\partial x}}_{\text{2te h. [illegible]}}$$

[illegible] Y, Z

[illegible] [[illegible] Lagr.] [illegible] 10, 11 [illegible] Kirchhoff

$$u\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial t} + u\frac{\partial \rho u}{\partial x} + \rho u\frac{\partial u}{\partial x} + u\frac{\partial \rho v}{\partial y} + \rho v\frac{\partial u}{\partial y} +$$

$= 0$

$$+ u\frac{\partial \rho w}{\partial z} + \rho w\frac{\partial u}{\partial z} = \rho X - \frac{\partial p}{\partial x}$$

[illegible] V u mult. Posten = 0 [illegible] Cont. [illegible]

$$\underbrace{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x}}_{\text{2te hydrod. [illegible]}}$$

3). $$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

4). $$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = Z - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

57. [illegible] $\rho = f(p)$ [illegible]

empirische [illegible]

Maxwell [illegible] kinet. Gastheorie [illegible]

1/e

[illegible] Hydrodyn. [illegible]

$$X = -\frac{\partial V}{\partial x}$$
$$Y = -\frac{\partial V}{\partial y}$$
$$Z = -\frac{\partial V}{\partial z}$$

V = Potential

[illegible]

$$u = \frac{\partial \varphi}{\partial x}$$
$$v = \frac{\partial \varphi}{\partial y}$$
$$w = \frac{\partial \varphi}{\partial z}$$

[illegible] φ = [illegible] Potential

(Helmholtz [illegible]

[illegible] Rotg. [illegible]

? [illegible] Rotg.

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$ [illegible]

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = [illegible]\ 0$$ [illegible] ? 0 [illegible]

[illegible]

[illegible] wirbellosen [illegible]

Wirbel- [illegible]

30/5

u, v, w | [illegible]

ρ [illegible] ...

p [illegible] ...

1. $\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$

[illegible]

2. $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x}$ [illegible]

3. $\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y}$ Euler [illegible]

4. — — — —

5. $\rho = f(p)$

$u = \frac{\partial\varphi}{\partial x}$ $\quad\varphi$ [illegible] Potential [illegible]

$v = \frac{\partial\varphi}{\partial y}$ $\quad$ I. [illegible]

$w = \frac{\partial\varphi}{\partial z}$

2a)

$$\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial x}\right) + \frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial\varphi}{\partial y}\frac{\partial^2\varphi}{\partial x\,\partial y} + \frac{\partial\varphi}{\partial z}\frac{\partial^2\varphi}{\partial x\,\partial z} = \backslash$$

[illegible] $X\,Y\,Z$ [illegible] Pot. [illegible]

$$X = -\frac{\partial V}{\partial x} \quad Y = -\frac{\partial V}{\partial y} \quad Z = -\frac{\partial V}{\partial z}$$

[illegible] 5 [illegible]: $\chi = \int_0^p \frac{dp}{\rho}$

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial \chi}{\partial x}$$

$\chi = f(p)$

$$= -\frac{\partial V}{\partial x} - \frac{\partial \chi}{\partial x}$$

$$\frac{\partial}{\partial x}\left[\frac{\partial\varphi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right]\right] = -\frac{\partial}{\partial x}\left[+V+\chi\right]$$

$u^2+v^2+w^2 = q^2 =$ [illegible]

$$\frac{\partial}{\partial x}\left[\frac{\partial\varphi}{\partial t} + \frac{1}{2} q^2 + V + \chi\right] = 0$$

[illegible] 3a), 4a) [illegible]

[illegible] x, y, z [illegible] t [illegible]

II. $\frac{\partial\varphi}{\partial t} + \frac{1}{2} q^2 + V + \chi = F(t)$

[illegible] ρ const. [illegible] incompress. [illegible]

$$\frac{\partial\rho}{\partial x},\ \frac{\partial\rho}{\partial y},\ \frac{\partial\rho}{\partial z},\ \frac{\partial\rho}{\partial t} = 0$$

[illegible] 1): $\rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y} + \rho\frac{\partial w}{\partial z} = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

I a) [illegible] φ [illegible]: $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0$

[illegible] H. [illegible] Int. [illegible] 2. Di[illegible]

[illegible] II: V [illegible] Pot.

[illegible] 3 V. u, v, w: [illegible] : φ

[illegible] Int. [illegible] φ [illegible]

[illegible] II: χ [illegible] 5 II

[illegible] I^a [illegible] Elektr., Magn., Diff. etc.

Riemann? [illegible]

stationäre Bewegungen:

[illegible] $f(x, y, z)$

$I^a)$ $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0$

$II^a)$ $\frac{1}{2}q^2 + V + \frac{p}{\rho} = C$

[illegible]

$\frac{p}{\rho} = C - V - \frac{1}{2}q^2$

[illegible]

$V = -gz$ [illegible]

$Z = -\frac{\partial V}{\partial z} = g$

$\frac{p}{\rho} = C + gz - \frac{1}{2}q^2$

$p = C\rho + \rho g z - \frac{1}{2}\rho q^2$

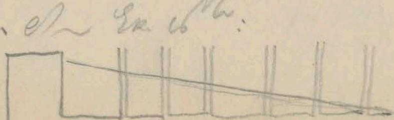


$p = A + \rho g z - \frac{1}{2}\rho q^2$

[illegible]

[illegible]

[illegible]:



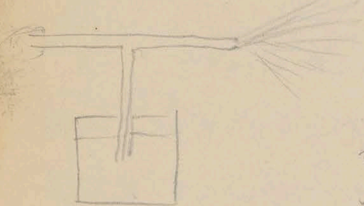
[illegible]

[illegible]

[illegible] ant. [illegible]

[illegible] 18

[illegible]



[illegible]

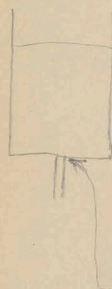
[illegible] injektor [illegible]

Locomotiven, [illegible]

Torricelli'sches Problem:

[illegible]

[illegible] Niveau [illegible]



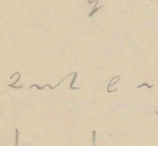
$p_0 = a$ = atm. [illegible]

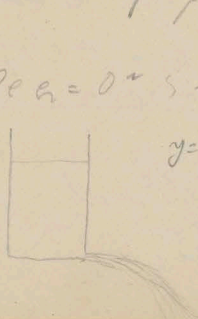
$p = a$

$0 = 0 + \rho g z - \frac{1}{2}\rho q^2$

$q = \sqrt{2gh}$ $z =$ [illegible]

[illegible] $= 0$ [illegible] atm. [illegible]

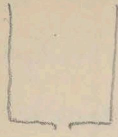




$y = qt \mid x = \frac{g}{2} t^2$

$t = \frac{x^2}{q^2} = \frac{2z}{g}$

$x^2 = \frac{2zq^2}{g} = 4zh$



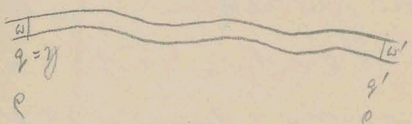
$Q = \mu\, B \cdot t \sqrt{2gh}$

[illegible]

$Q = m\, B t \sqrt{2gh} \quad m < 1 \;\Big|\; 0.68 - 0.74 \quad 0.61$

[illegible]

[illegible] [Vena contracta] [illegible]



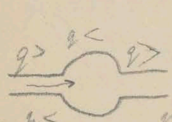
[illegible]

[illegible]

$\omega q \rho = \omega' q' \rho$

$q : q' = \omega' : \omega$ [illegible]

[illegible]



[illegible]

p< [illegible]

? I^e ℓ init.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$b_1 q_1 = b_2 q_2$

$q_1 : q_2 = b_2 : b_1$ [illegible]

[illegible]

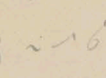
[illegible]

$$\frac{\partial\varphi}{\partial x} dx + \frac{\partial\varphi}{\partial y} dy + \frac{\partial\varphi}{\partial z} dz = u\,dx + v\,dy + w\,dz$$

$$\varphi = \int u\,dx + v\,dy + w\,dz$$

$$= \int u_s\,ds$$

[illegible]



[illegible]

[illegible] const. [illegible] const. [illegible]

[illegible]



[illegible]

[illegible] verif. [illegible] φ [illegible] const. [illegible]

Dirichlets [illegible] f. theorie

[illegible]

1) $\Delta\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0$

2) $\Delta\varphi + \frac{1}{2} q^2 + V + \int \frac{dp}{\rho} = 0$

$\frac{\partial\varphi}{\partial x} = u \quad \frac{\partial\varphi}{\partial y} = v \quad \frac{\partial\varphi}{\partial z}\, w$

$q^2 = u^2 + v^2 + w^2$

Helmholtz [illegible]

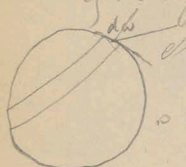
H. [illegible]

$\varphi = \int_a^m u\,dx + v\,dy + w\,dz$

[illegible]

[illegible]

[illegible]



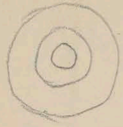
[illegible] $d\omega \frac{\partial\varphi}{\partial r}$

$\iint d\omega \frac{\partial\varphi}{\partial r} = 0$ [illegible] Green's [illegible]

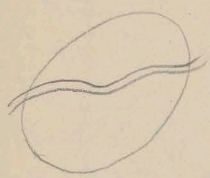
[illegible] Sound [illegible]

$\iint \frac{\partial}{\partial r} \varphi\, d\omega = 0 = \frac{\partial}{\partial r} \int \varphi\, d\omega$

$\frac{\Sigma f m}{\Sigma m} =$ [illegible] $\quad \frac{\partial}{\partial r} \varphi_m = 0$ | [illegible]



[illegible]



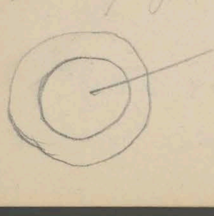
[illegible]

[illegible] φ_1, φ_2 [illegible] $\varphi_2 - \varphi_1$ [illegible]

[illegible]

[illegible] Riemann [illegible]

[illegible]



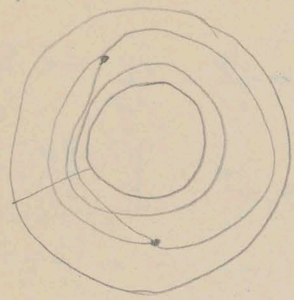
[illegible] – Curve 98

[illegible]

[illegible]

[illegible] Int. [illegible]

[illegible]



$\varphi = \varphi' + k\,n$

κ = Periode

z.B. $\arcsin = \int \frac{dx}{\sqrt{1-x^2}}$

$/x=1$ [illegible]

[illegible] Elektr.

[illegible]

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0$$

$$\frac{\partial\varphi}{\partial x} = u \qquad \frac{\partial\varphi}{\partial y} = v$$

$$u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x}$$

$$\frac{\partial\psi}{\partial y} = \frac{\partial\varphi}{\partial x} \qquad \frac{\partial\psi}{\partial x} = -\frac{\partial\varphi}{\partial y}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$

$$\frac{\partial\varphi}{\partial x}\frac{\partial\psi}{\partial x} + \frac{\partial\varphi}{\partial y}\frac{\partial\psi}{\partial y} = 0$$

$\varphi(x,y)$

$\psi(x,y)$

[illegible]

$$f(x+yi) = \varphi(x,y) + i\psi(x\,y)$$

$$f' = \frac{\partial\varphi}{\partial x} + i\frac{\partial\psi}{\partial x}$$

$$f'i = \frac{\partial\varphi}{\partial y} + i\frac{\partial\psi}{\partial y}$$

$$i\frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial x} = \frac{\partial\varphi}{\partial y} + i\frac{\partial\psi}{\partial y}$$

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y} \quad\Big|\quad \frac{\partial\varphi}{\partial y} = -\frac{\partial\psi}{\partial x}$$

[illegible]

$$\underbrace{f(x+yi)}_{z} = w \qquad w = f(z)$$

$$dx\,\frac{\partial w}{\partial z} =$$

$$\frac{dw}{dz} = \frac{\left(\frac{\partial\varphi}{\partial x} + i\frac{\partial\psi}{\partial x}\right) + \left(\frac{\partial\varphi}{\partial y} + i\frac{\partial\psi}{\partial y}\right)dy}{dx + i\,dy}$$

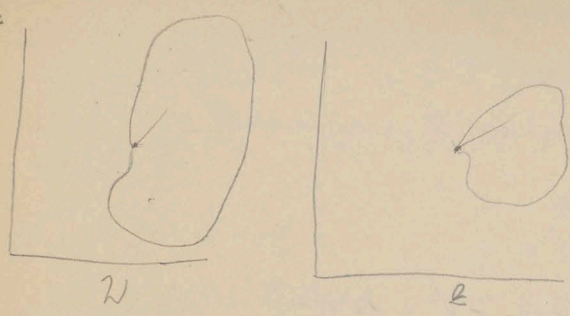
$$\frac{\partial\varphi}{\partial x} + i\frac{\partial\psi}{\partial x} = \left(\frac{\partial\varphi}{\partial y} + \frac{\partial\psi}{\partial y}\right)$$

?

$$\frac{\partial\varphi}{\partial y} = \frac{\partial\psi}{\partial x} \qquad \frac{\left(\frac{\partial\varphi}{\partial x} + i\frac{\partial\varphi}{\partial x}\right)(dx + i\,dy)}{dx + i\,dy}$$

$$\frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y}$$

$w =$



$dw = \frac{\partial w}{\partial z} dz$

$F(x, y, z)$

$w = z^{\frac{1}{2}} = [x + yi]^{\frac{1}{2}}$

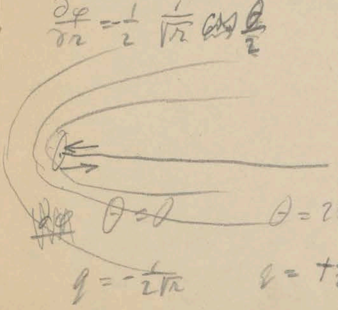
$= [r(\cos\theta + i \sin\theta)]^{\frac{1}{2}}$

$= \sqrt{r}\left[\cos\frac{\theta}{2} + i \sin\frac{\theta}{2}\right]$

$\varphi = \sqrt{r} \cos\frac{\theta}{2}$ $\qquad S = \frac{\partial \varphi}{\partial s}$

$\psi = \sqrt{r} \sin\frac{\theta}{2}$

$\frac{\partial \varphi}{\partial r} = \frac{1}{2} \frac{1}{\sqrt{r}} \cos\frac{\theta}{2}$



[illegible]

$\theta = 0$ $\qquad \theta = 2\pi$ [illegible]

$q = -\frac{1}{2\sqrt{r}}$ $\qquad q = +\frac{1}{2\sqrt{r}}$ [illegible]

Längs Curve

$\psi = \text{const.}$

$z^2 = r^2 \sin^2\frac{\theta}{2} = r[1-\cos\theta]$

~~$+x^2$~~

$(c^2+x)^2 =$

$c^4+2c^2x+x^2 = x^2+y^2$

Parabel

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[illegible]

[illegible]

[illegible]

$u = \frac{\partial\varphi}{\partial x}$ $v = \frac{\partial\varphi}{\partial y}$ $w = \frac{\partial\varphi}{\partial z}$

1). $\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$ Continuitätsgl.

2). $\frac{\partial\varphi}{\partial t} + \frac{1}{2}\left(u^2+v^2+w^2\right) + \int\frac{dp}{\rho} + V = 0$ (the term $\frac{1}{2}(u^2+v^2+w^2)$ is struck through)

3). $\int\frac{dp}{\rho} = ?$

[illegible] 2 — [illegible]

$\rho = \rho_0(1+\sigma)$ $\sigma =$ [illegible] $= \frac{\rho-\rho_0}{\rho_0}$

[illegible] σ [illegible]

$\frac{\partial\rho}{\partial t} = \rho_0\frac{\partial\sigma}{\partial t}$

$\rho u = \rho_0(1+\sigma)u = \rho_0 u + \rho_0\sigma u$ [illegible]

[illegible] ×

$\rho u = \rho_0 u$

$\rho v = \rho_0 v$

$\rho w = \rho_0 w$

$\rho_0 = \frac{1}{770}/_5$

1a). $\rho_0\frac{\partial\sigma}{\partial t} + \rho_0\frac{\partial u}{\partial x} + \rho_0\frac{\partial v}{\partial y} + \rho_0\frac{\partial w}{\partial z} = 0$

$\frac{\partial\sigma}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$\frac{\partial \sigma}{\partial t} + \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0$$

1a) $$\frac{\partial \sigma}{\partial t} + \Delta \varphi = 0$$

[illegible]

$$\frac{\partial \varphi}{\partial t} = -\int \frac{dp}{\rho}$$

[illegible] $\quad p = \frac{p_0}{\rho_0}\rho$

$$dp = \frac{p_0}{\rho_0} d\rho$$

$$\frac{dp}{\rho} = \frac{p_0}{\rho_0}\frac{d\rho}{\rho} = \frac{p_0}{\rho_0} d(\log \rho)$$

$$\int \frac{dp}{\rho} = \frac{p_0}{\rho_0} \log \rho$$

2a') $$\frac{\partial \varphi}{\partial t} = -\frac{p_0}{\rho_0}\log \rho$$

$$\frac{\partial^2 \varphi}{\partial t^2} = -\frac{p_0}{\rho_0}\frac{\partial \rho}{\partial t}\frac{1}{\rho} = -\frac{p_0}{\rho_0}\frac{1}{\rho}\rho_0\frac{\partial \sigma}{\partial t}$$

$$-\frac{\partial \sigma}{\partial t} = \Delta \varphi$$

3). $$\frac{\partial^2 \varphi}{\partial t^2} = \frac{p_0}{\rho_0}\Delta \varphi = a^2 \Delta \varphi$$

2B. [illegible]

[illegible] $x=0$ [illegible]

$$\frac{\partial^2 \varphi}{\partial t^2} = a^2 \frac{\partial^2 \varphi}{\partial x^2}$$

[illegible]

Lösung v. d'Alembert: [illegible]

f. [illegible]:

$$\varphi = f(x \pm at)$$

$$\frac{\partial \varphi}{\partial t} = \pm f'(x \pm at)\, a \qquad \frac{\partial \varphi}{\partial x} = f'(x \pm at)$$

$$\frac{\partial^2 \varphi}{\partial t^2} = f''(x \pm at)\, a^2 \qquad \frac{\partial^2 \varphi}{\partial x^2} = f''(x \pm at)$$

$$\varphi = f(x - at) + F(x + at)$$ [illegible] (partiell)

2 [illegible]

[illegible]: $\varphi = f(x - at)$

$t = 0$, $\varphi = f(0) = C$

$x > 0$ $\varphi = 0$

$\varphi = f(0)$ $(x - at) = 0$ [illegible]

$x = at$

[illegible] a = [illegible]

[illegible]

$$a = \sqrt{\frac{p_0}{\rho_0}} \qquad \rho_0 = \frac{1}{770}\,g \quad g \cdot 0 \quad 980\ \text{cm}$$

$p_0 = 1033\ g\ g' \times$

```
   1033
    9297
     8264
  1012 34
  708638
   708 638
√7|7 9|5 0|1 8|00 = 278 00 = 278 m
 3  7 7           : 47
   50 00          : 54
```

$\frac{p_0}{\rho_0}$

Laplace [illegible] $\sim p^{1}/\rho$

$$\frac{\partial \sigma}{\partial t} + \frac{\partial u}{\partial x} + \ldots = 0$$

$$1^{a})\ \frac{\partial \sigma}{\partial t} + \Delta\varphi = 0$$

Laplace

$$\frac{\partial \varphi}{\partial t} = -\int \frac{dp}{\rho}$$ [illegible]

[illegible]

$$p v = p_0 v_0 \left(1 + \frac{t}{273}\right)$$

$$\frac{p_0 v_0}{273} \overbrace{(273 + t)}^{T} = R = p_n v_n$$

$$p v = R T$$

[illegible]

const. [illegible]

$$p\,dv = R\,dT$$

$$dQ = c\,dT + A.\,p\,dv$$

$$dQ = (c + AR)\,dT \quad$$ [illegible] $$C = c + AR$$ [illegible]

$$p\,dv + v\,dp = R\,dT$$

$$dT = \frac{p}{R}\,dv + \frac{v}{R}\,dp$$

$$dQ = \frac{1}{R}\left[p\,dv(c + AR) + c\,v\,dp\right]$$ adiab. [illegible] $dQ = 0$

$$0 = c\,v\,dp + C\,p\,dv$$

$$0 = c\,\frac{dp}{p} + C\,\frac{dv}{v}$$

$$= c\,d\log p + C\,d\log v$$

$$p^{c}\, v^{C} = \text{const.} = M$$

$0 =$

$p \cdot v^{\frac{C}{c}} = n \qquad \frac{C}{c} = 1{,}4$

$\frac{C}{c} = \gamma \qquad p = n\rho^{\gamma}$

$\frac{C}{c} = 1{,}66$ einatomig

$= 1{,}41$ zwei —

$= 1{,}2$ drei —

$dp = \gamma n \rho^{\gamma-1}$

$\frac{dp}{\rho} = \gamma n \rho^{\gamma-2}$

$\int \frac{dp}{\rho} = \frac{\gamma n}{\gamma-1} \rho^{\gamma-1}$

$\frac{\partial \sigma}{\partial t} + \Delta\varphi = 0$

$\frac{\partial \varphi}{\partial t} + \frac{\gamma n}{\gamma-1} \rho^{\gamma-1} = 0 \qquad \rho = \rho_0(1+\sigma)$

$\frac{\partial^2 \varphi}{\partial t^2} + \frac{\gamma n}{\gamma-1} \rho_0^{\gamma-2} \frac{\partial \sigma}{\partial t} \cdot \rho_0 = 0$

$\frac{\partial^2 \varphi}{\partial t^2} = \gamma n \rho_0^{\gamma-1} \Delta\varphi = a^2 \Delta\varphi$

$a^2 = \gamma n \rho_0^{\gamma-1} \qquad | \; a^2 = \frac{p_0}{\rho_0}$ [illegible]

$\frac{\partial^2 \varphi}{\partial t^2} = a^2 \frac{\partial^2 \varphi}{\partial x^2}$ — $\varphi = f(x - at)$

$a = \sqrt{n\gamma \rho_0^{\gamma-1}} \qquad p = n\rho^{\gamma}$

$= \sqrt{\frac{p_0}{\rho_0}\gamma} \qquad \frac{p_0}{\rho_0} = n\rho_0^{\gamma-1}$

$\sqrt{\frac{p_0}{\rho_0}} = 278\,m \qquad \sqrt{1{,}41} \neq 1{,}2$

278
$\;\;556$
$\overline{333{,}6}$

[illegible] | $\lambda =$ [illegible]

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$\varphi \qquad \frac{\partial\varphi}{\partial x} = u_x \;\Big|\; \frac{\partial\varphi}{\partial y} = u_y \;\Big|\; \frac{\partial\varphi}{\partial z} = w_z$

$\varphi(x\,y\,z\,t) = 0$

$\frac{\partial^2\varphi}{\partial t^2} = a^2 \Delta\varphi \qquad \Delta\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}$

$a = \sqrt{\left(\frac{p}{\rho}\right)\gamma} \qquad \gamma = \frac{C}{c} = 1{,}4$

[illegible] [[illegible]] [illegible]

$\varphi(r\,t)$



$r^2 = x^2 + y^2 + z^2$

$\frac{\partial\varphi}{\partial x} = \frac{\partial\varphi}{\partial r}\frac{dr}{dx}$

$2r\frac{\partial r}{\partial x} = 2x$

$\frac{\partial\varphi}{\partial x} = \frac{\partial\varphi}{\partial r}\frac{x}{r}$

$\frac{\partial^2\varphi}{\partial x^2} = \frac{\partial^2\varphi}{\partial r^2}\frac{x^2}{r^2} + \frac{\partial\varphi}{\partial r}\frac{1}{r} - \frac{\partial\varphi}{\partial r}\frac{x}{r^2}\frac{x}{r}$

$\frac{\partial^2\varphi}{\partial y^2} = \frac{\partial^2\varphi}{\partial r^2}\frac{y^2}{r^2} + \frac{\partial\varphi}{\partial r}\frac{1}{r} - \frac{\partial\varphi}{\partial r}\frac{y^2}{r^3}$

$\frac{\partial^2\varphi}{\partial z^2} = \frac{\partial^2\varphi}{\partial r^2}\frac{z^2}{r^2} + \frac{\partial\varphi}{\partial r}\frac{1}{r} - \frac{\partial\varphi}{\partial r}\frac{z^2}{r^3}$

$\Delta\varphi = \frac{\partial^2\varphi}{\partial r^2} + \frac{3}{r}\frac{\partial\varphi}{\partial r} - \frac{\partial\varphi}{\partial r}\frac{1}{r}$

$\Delta\varphi = \frac{\partial^2\varphi}{\partial r^2} + \frac{2}{r}\frac{\partial\varphi}{\partial r}$

$\frac{\partial^2\varphi}{\partial t^2} = a^2\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{2}{r}\frac{\partial\varphi}{\partial r}\right)$ f. D. S integr. [illegible]

[illegible] Fourier

$r\frac{\partial^2\varphi}{\partial t^2} = a^2\underbrace{\left(r\frac{\partial^2\varphi}{\partial r^2} + 2\frac{\partial\varphi}{\partial r}\right)}_{\frac{\partial^2}{\partial r^2}(r\varphi)}$

$$\frac{\partial}{\partial r}(r\varphi) = \varphi + r\frac{\partial \varphi}{\partial r}$$

$$\frac{\partial^2 (r\varphi)}{\partial r^2} = \frac{\partial \varphi}{\partial r} + \frac{\partial \varphi}{\partial r} + r\frac{\partial^2 \varphi}{\partial r^2} = r\frac{\partial^2 \varphi}{\partial r^2} + 2\frac{\partial \varphi}{\partial r}$$

$$r\frac{\partial^2 \varphi}{\partial t^2} = a^2 \frac{\partial^2 (r\varphi)}{\partial r^2}$$

$\frac{\partial^2 (r\varphi)}{\partial t^2}$ [illegible]

$$\frac{\partial^2 (r\varphi)}{\partial t^2} = a^2 \frac{\partial^2 (r\varphi)}{\partial r^2} \qquad \Big| \text{[illegible]} \ \frac{\partial^2 \varphi}{\partial t^2} = a^2 \frac{\partial^2 \varphi}{\partial x^2}$$

$$\varphi = f(x - at)$$

$$\varphi = \frac{1}{r} F(r - at)$$

$$r\varphi = F(r - at)$$

$$\frac{\partial \varphi}{\partial r} = \frac{F'(r-at)}{r} - \frac{1}{r^2}F(r-at)$$

$\left(\frac{\partial \varphi}{\partial r}\right)^2 = u^2 =$ [illegible] $=$ Intens. [illegible]

[illegible]

[illegible] Problems [illegible] Riemann II.

[illegible]

[illegible] Gerstner [Kirchhoff]

[illegible] Rankine [illegible] II

[illegible] Weber

[illegible]

Helmholtz [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

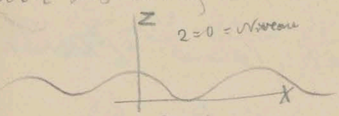
1). $\Delta\varphi = 0$

2). $\frac{\partial\varphi}{\partial t} + ~~\frac{1}{2}q^2~~ + V + \int\frac{dp}{\rho} = H$ [illegible]

↑ [illegible] $= \frac{p}{\rho}$ [illegible] incompr.

V [illegible]

[illegible] Vert. [illegible]



[illegible] Cylinder [illegible]

[illegible]

$\varphi = F(x, z, t)$

$X = -\frac{\partial V}{\partial x}$ $Z = -\frac{\partial V}{\partial z}$

$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2} = 0$

$Z = -g$

$\frac{\partial\varphi}{\partial t} + \frac{p}{\rho} + gz = H$

1). $\varphi = f(x + zi)$

? [illegible] [1] [illegible] $= 0$

$z = -h \,|$

$\frac{\partial \varphi}{\partial z} = 0$

2). [illegible]

[illegible]

$\frac{\partial p}{\partial t} = 0$

$\frac{\partial^2 \varphi}{\partial t^2} + \cancel{\frac{\partial p}{\partial t}\frac{1}{\rho}} + g \underbrace{\frac{\partial z}{\partial t}} = 0$

[illegible] $Z = \frac{\partial \varphi}{\partial z}$

$z = 0 \,\Big|\quad \frac{\partial^2 \varphi}{\partial t^2} = -g \frac{\partial \varphi}{\partial z}$



[illegible]: $\varphi = f(z) \cos n(x - ct)$

$\cancel{\frac{d^2 f}{dz^2} \cos} \quad + f(z)\, n^2 \cos \qquad = 0$

$\frac{d^2 f}{dz^2} = n^2 f(z) \qquad$ [illegible] $a e^{nz} + b e^{-nz}$

$z = -h \,\Big|\; \frac{\partial \varphi}{\partial z} = 0$

$\frac{\partial \varphi}{\partial z} = \frac{df(z)}{dz} \cos \;\; = 0 \qquad \frac{df}{dz} = a n e^{nz} - b n e^{-nz}$

$d\,\frac{f(z)}{dz} = 0 \quad \Big|^{z=-h} \quad 0 = a n e^{-nh} - b n e^{nh}$

$a = A e^{nh} \qquad b = A e^{-nh}$

$$f(z) = A\left[e^{n(h+z)} + e^{-n(h+z)}\right]$$

$$\varphi = A\left[e^{n(z+h)} + e^{-n(z+h)}\right]\cos n(x-ct)$$

$$\frac{\partial^2\varphi}{\partial t^2} = -A\left[e^{n(z+h)} + e^{-n(z+h)}\right]\cos n(x-ct)\, n^2c^2$$

$$+ An\left[e^{n(z+h)} - e^{-n(z+h)}\right]\cos n(x-ct)$$

$\cos(nx - nct)$

$nc \sin$

$-\cos \quad - \quad n^2c^2$

$z=0$

$$nc^2 = \frac{e^{nh} - e^{-nh}}{e^{nh} + e^{-nh}}$$

$$c^2 = \frac{g}{n}\,\frac{e^{nh} - e^{-nh}}{e^{nh} + e^{-nh}}$$

$$\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \qquad \lambda = cT$$

$$\cos\frac{2\pi}{\lambda}\left(x - \frac{t\lambda}{T}\right) \qquad \frac{\lambda}{T} = c$$

$$\cos\frac{2\pi}{\lambda}\left(x - \frac{ct}{T}\right)$$

$$n = \frac{2\pi}{\lambda}$$

2,

I $h =$ [illegible]

$$c^2 = \frac{g\lambda}{2\pi} \qquad c = \sqrt{\frac{g\lambda}{2\pi}}$$

[illegible]

II $h =$ [illegible]

$$\frac{1+nh \;-\; 1+nh}{1+nh \;+\; 1-nh} = nh$$

$$c^2 = \frac{g}{n} \cdot nh = gh$$

$$c = \sqrt{gh}$$

[illegible] Gerstner, Rankine ‖

[illegible] Riemann Thomson . . . .

[illegible]

[illegible]

I. $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2} = 0$

[illegible]

II. $\frac{\partial\varphi}{\partial z} = 0$ III $\frac{\partial\varphi}{\partial t} + gz + \frac{p}{\rho} = const.$

$$\varphi = A\left(e^{m(z+h)} + e^{-m(z+h)}\right)\cos n(x-ct)$$

$$\frac{2\pi}{n} = \lambda \qquad n = \frac{2\pi}{\lambda}$$

$$c^2 = \frac{g}{n}\,\frac{e^{nh}+e^{-nh}}{e^{nh}-e^{-nh}}$$

2π [illegible] $c = \sqrt{\frac{g}{n}}$

[illegible] $c = \sqrt{gh}$

? [illegible]

? [illegible]

$$\frac{\partial\varphi}{\partial t} = +A\left(e^{n(z+h)} + e^{-n(z+h)}\right)\sin n(x-ct)\,nc = -gz + C$$

z = surf. [illegible]

1). $z = -\frac{ncA}{g}\left[e^{nh} + e^{-nh}\right]\sin n(x-ct)$ [illegible]

? [illegible]

2). $\frac{\partial\xi}{\partial t} = \frac{\partial\varphi}{\partial x} = -A\left(e^{n(z+h)} + e^{-n(z+h)}\right)\sin n(x-ct)\cdot n$

3). $\frac{\partial\zeta}{\partial t} = \frac{\partial\varphi}{\partial z} = An\left(e^{n(z+h)} - e^{-n(z+h)}\right)\cos n(x-ct)$

$$\xi = -\frac{nA}{nc}\left(e^{n(z+h)} + e^{-n(z+h)}\right)\cos n(x-ct)$$

$$\zeta = -\frac{nA}{nc}\left(e^{n(z+h)} - e^{-n(z+h)}\right)\sin n(x-ct)$$

$$\frac{\xi^2}{\left[\frac{A}{c}\left(e^{n(z+h)}+e^{-n(z+h)}\right)\right]^2}+\frac{\zeta^2}{\left[\frac{A}{c}\left(e^{n(z+h)}-e^{-n(z+h)}\right)\right]^2}=1$$

$$\frac{b}{a}=\frac{e^{n(z+h)}-e^{-n(z+h)}}{e^{n(z+h)}+e^{-n(z+h)}}$$

[illegible] $z=0$

$b \neq a$ [illegible]

[illegible] Ellipsen

[illegible] horiz.

Wilhelm Weber [illegible] experiment. [illegible]

[illegible] Energie [illegible] kinet. Energie,

[illegible] potent. Energie, [illegible]

[illegible] elast. [illegible] elektr. [illegible]

[illegible] elektrostat. [illegible] elektrodyn.

[illegible] pot. E. = kin. En.

[illegible] kin. [illegible] pot. E. [illegible] E. ?

$$\int \rho g z \, dx$$



[illegible] $\frac{z}{2}$

$$\int_0^\lambda \rho g \frac{z^2}{2} dx$$

$$\rho g \frac{m^2 A^2}{c^2}\left(e^{nh}+e^{-n}\right)^2 \int_0^{2\pi} \sin^2(x-ct)$$

$\frac{\lambda}{2}$ + [illegible] Pot. En. $E_v = \rho \frac{n^2 A^2 \lambda}{4g}\left(e^{nh}+e^{-nh}\right)^2$

$\rho\, dx\, dz$ $\qquad x \quad 0 - \lambda$

$\qquad\qquad z \quad -0 - h$

kin. Energie =

$$\int_0^{\lambda}\int_0^{h}\frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right]\rho\, dx\, dz$$

$$= \frac{n^2A^2\rho}{2}\int_0^{-h}\left[e^{2n(z+h)} + e^{-2n(z+h)} \cancel{+2}\right]\int_0^{\lambda}\underbrace{\sin^2 n(x-ct)}_{\frac{\lambda}{2}} +$$

$$+\frac{n^2A^2\rho}{2}\underbrace{\int_0^{-h}\left[e^{2n(z+h)} + e^{-2n(z+h)} \cancel{-2}\right]}_{Z}\int_0^{\lambda}\underbrace{\cos^2 n(x-ct)}_{\frac{\lambda}{2}} =$$

$$= \frac{n^2A^2\rho}{2}\lambda\, Z$$

$$Z = \frac{1}{2n}\left(e^{2n(z+h)} - e^{-2n(z+h)}\right)\Bigg|_{z=0}^{z=-h}$$

$$\mathfrak{L} = \frac{n^2A^2\rho\lambda}{4n}\left(e^{2nh} \cancel{- e^{-2nh}}\right) \quad |^{\circ}/\infty$$

$$\mathfrak{L}_p = \frac{n^2\rho\, c^2A^2\lambda}{4g}\, e^{2nh} \qquad c^2 = \frac{g}{n}$$

$$\mathfrak{L}_v = \frac{n^2\rho\, g A^2\lambda}{4ng}\, e^{2nh} = \mathfrak{L}_p$$

– William Thomson

[illegible]

[illegible]

[illegible shorthand] III [illegible shorthand]

$$\frac{\partial\varphi}{\partial t} + gz + \frac{p}{\rho} \qquad p = H\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$z = f(xy)$$

$$p = H\left[\frac{d^2 z}{dx^2} + \frac{d^2 z}{dy^2}\right] = H\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$$

$$\frac{\partial\varphi}{\partial t} + gz + \frac{H}{\rho}\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) = \text{const.}$$

$$\frac{\partial z}{\partial t} = \frac{\partial\varphi}{\partial t}$$

$$\frac{\partial^2\varphi}{\partial t^2} + g\frac{\partial\varphi}{\partial z} + \frac{H}{\rho}\left(\frac{\partial^3\varphi}{\partial x^2\partial z} + \frac{\partial^3\varphi}{\partial y^2\partial z}\right) = 0$$

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 \qquad -\frac{\partial^3\varphi}{\partial z^3}$$

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = -\frac{\partial^2\varphi}{\partial z^2}$$

$$\text{II.}\ \frac{\partial^2\varphi}{\partial t^2} + g\frac{\partial\varphi}{\partial z} + \frac{H}{\rho}\frac{\partial^3\varphi}{\partial z^3} = 0 \ \Big|\ z = 0$$

$$\left.\begin{array}{l} -A\left[e^{n(z+h)} + e^{-n(z+h)}\right]\cos n(x-ct) \cdot n^2c^2 \\ +gAn\left[e^{n(z+h)} - e^{-n(z+h)}\right]\cos n(x-ct) \\ -\frac{H}{\rho}An^3\left[e^{n(z+h)} - e^{-n(z+h)}\right]\cos n(x-ct) \end{array}\right\} = 0$$

$$n^2c^2\left(e^{nh} + e^{-nh}\right) = \left[gn + \frac{H}{\rho}n^3\right]\left[e^{nh} - e^{-nh}\right]$$

$$c^2 = \left[\frac{g}{n} + \frac{H}{\rho}n\right]\frac{e^{nh} - e^{-nh}}{e^{nh} + e^{-nh}} \qquad n = \frac{2\pi}{\lambda}$$

[illegible]

$$c^2 = \left[\frac{g\lambda}{2\pi} + \frac{2\pi H}{\rho\lambda}\right]\frac{e^{nh} - e^{-nh}}{\ldots} \ \Bigg\|\ c^2 = \frac{g\lambda}{2\pi}\frac{e^{nh} + e^{-nh}}{e^{nh} - e^{-nh}}$$

[illegible]

λ_1, [illegible]

λ_2 [illegible]

$$c^2 = \frac{2\pi H}{\rho\lambda} \, \frac{e^{nh} - e^{-nh}}{e^{nh} + e^{-nh}}$$

[illegible] H [illegible]

[illegible]

$$c^2 = 2\sqrt{\frac{gH}{\rho}} + \left[\sqrt{\frac{g\lambda}{2\pi}} - \sqrt{\frac{2\pi H}{\rho\lambda}}\right]^2$$

[illegible]

$$c = 2\sqrt{\frac{gH}{\rho}}$$

[illegible]

[illegible]

$$U^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi h}{\rho\lambda}\right)\frac{e^{mh}-e^{-mh}}{e^{mh}+e^{-mh}}$$ [illegible]

$$U_m^2 = 2\sqrt{\frac{gh}{\rho}} + \left[\sqrt{\frac{g\lambda}{2\pi}} - \sqrt{\frac{2\pi h}{\rho\lambda}}\right]^2$$

Minimum [illegible]:

$$\sqrt{\frac{g\lambda_m}{2\pi}} = \sqrt{\frac{2\pi h}{\rho\lambda_m}}$$ $h = 0{\cdot}074$ g/cm² [illegible]

g [illegible]

$$U^2 = 2\sqrt{\frac{gh}{\rho}}$$

$0{\cdot}074 \cdot g^2$ $\sqrt{74{,}000} = 272$

340 : 47

1100 : 58

$\sqrt{4.55} = 2.1$ cm

5 2 : 4

Russel ?

[illegible] Helmholtz [illegible]

$$g\rho\lambda^2 = 4\pi^2 h$$

$$\lambda = 2\pi\sqrt{\frac{h}{\rho g}}$$

$$= 2\pi\sqrt{0{\cdot}0740} = 2\pi \cdot 0{\cdot}27 = 6{\cdot}28 \cdot 0{\cdot}27 = 1{\cdot}7 \text{ cm}$$

1256
4396
1·6956

g/cm $U^2 = \frac{g\lambda}{2\pi}$

$$\lambda = \frac{2\pi U^2}{g}$$

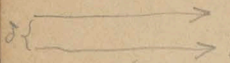
~~$\frac{d^2\phi}{dx^2}$~~ $\left(\frac{d\phi}{dx}\right)$ [illegible]

Helmholtz [illegible]

Er km ?

Velocità ... 2 Coeff. 1 ... 2 ... Deformation

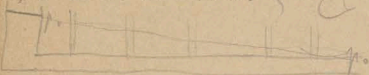
... ideale ...



$u > u'$

$\frac{u+u'}{\delta}$

Newton ...





$\frac{\partial u}{\partial r}$? $\quad + 2\pi r\lambda \cdot \frac{\partial u}{\partial r}$ $\quad + 2\pi\lambda\left(r\frac{\partial u}{\partial r}\right) - 2\pi\lambda\left(r\frac{\partial u}{\partial r}\right)'$

$r\frac{\partial u}{\partial r} = f(r)$

$\left(r\frac{\partial u}{\partial r}\right)' = f(r+dr) = f(r) + \frac{\partial f}{\partial r}dr +$



$= 2\pi\mu\left[\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \cdot dr\right] = p\,2\pi r\,dr$

$- p'\,2\pi r\,dr$

$p = f(x)$

$p' = f(x+dx) = p + \lambda\frac{\partial p}{\partial x}$

$= -2\pi r\,dr\,\lambda\frac{\partial p}{\partial x}$

$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{\partial p}{\partial x}$

$\frac{\partial p}{\partial x}$ = const. B.J. [illegible]

$a = -\frac{p_1 - p_0}{l}$ $\qquad u = \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right) = ar$

$$r \frac{\partial u}{\partial r} = \frac{ar^2}{2} + b$$

$$\frac{\partial u}{\partial r} = a\frac{r}{2} + \frac{b}{r}$$

$$u = a\frac{r^2}{4} + b \log r + c$$

$b = 0$

$$u = \frac{ar^2}{4} + c$$

$u = -\frac{p_1 - p_0}{\mu l}\frac{r^2}{4} + c$

[illegible] $\qquad r = R \mid u = u_0$ $\qquad$ u [illegible]

$C = \frac{R^2}{4} - \frac{r^2}{4}$ [illegible]



$\int_0^R 2\pi r\,dr = q$ $\quad$ u as const. [illegible]

Tyndall [illegible]

$$q = 2\pi \frac{p_1 - p_0}{4\mu l}\int_0^R \left(\frac{R^2}{2} - \frac{r^2}{4}\right) r\,dr$$

$$= \frac{p_1 - p_0}{2\mu l}\pi\left(\frac{R^2 r^2}{2} - \frac{r^4}{4}\right)$$

$q = \pi \frac{p_1 - p_0}{8\mu l} R^4$

Helvetia [illegible] = 24°C $\quad$ 0·014061

$u = \frac{a}{[illegible]}\left(\frac{R^4}{4} - \frac{r^2}{4}\right)$ [illegible]

